LABORATORY 10 – Transient Response in 1st And 2nd Order Circuits

Lab Goals

In this lab you will design, construct, and test a number of circuits with one or two energy-storing elements. The goal of the lab is to characterize and understand the transient response of these circuits.

Definitions

Overshoot – the maximum amount a signal (usually a voltage) exceeds its steady state value as the circuit transitions to equilibrium. For example, if the maximum voltage is V_m and the steady-state voltage is V_0, the percentage overshoot is:

$$OS = (V_m / V_0 - 1) * 100\,\%.$$

Ringing – the oscillation phenomena that occurs in an underdamped circuit as it approaches equilibrium.

Circuit Analysis

In this section we are going to briefly **REVIEW** the transient response for several simple circuits. We are always going to use the transfer function approach to solve for the homogeneous solutions and will leave everything in complex frequency notation: $s = j\omega \sim \partial / \partial t$.

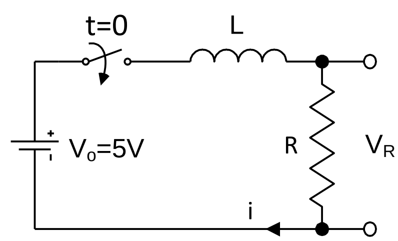

Figure 11.1 A simple RL circuit.

The first circuit we will analyze is the RL series combination shown in Fig. 11.1. KVL for $t > 0$ yields

$$V_o = L\frac{di}{dt} + Ri(t) \rightarrow V_o = (Ls + R)\hat{I}\ .$$

Thus, the particular solution is

$$i_p(t) = V_o / R = 5 / R \text{ (Amps)}$$

and the homogeneous solution is

$$i_h(t) = i_0 e^{-Rt/L}\ .$$

The initial condition i_0 is found from the total solution $i(t) = i_h(t) + i_p(t)$, after using the continuity of inductor current principle and the fact that the switch is open for $t < 0$. The final answer is:

$$i(t) = \frac{5}{R}(1 - e^{-Rt/L}). \text{ (A)}$$

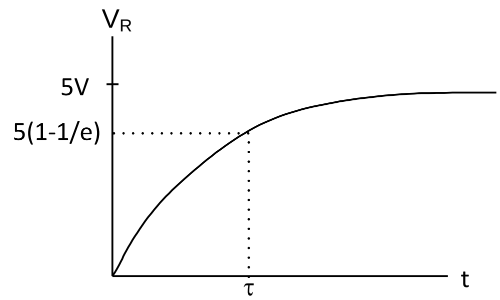

Figure 11.2 The transient response for the1st-order circuit.

The corresponding voltage across the resistor is plotted in Fig. 11.2. How could we determine the inductance L without an LC meter? We know the starting and ending values of the resistor voltage, so we can pick a point in between, measure the voltage at that time and select L so that the above equation agrees. For example, if we pick the point where the voltage has achieved $(1 - e^{-1}) = 63.2\%$ of its final value, the time τ that this voltage occurs can be used to find L (assuming a DMM was used to get R):

$$L = R\tau.$$

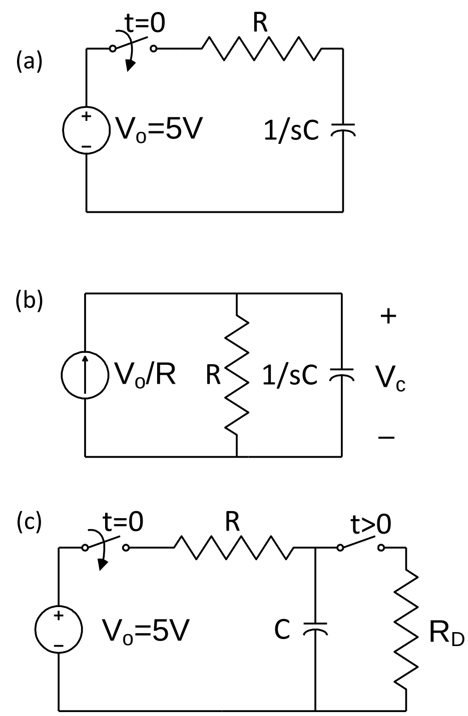

Figure 11.3 A simple RC circuit.

The second circuit we'll analyze is the RC series circuit shown in Fig. 11.3a. It may be more convenient to convert the nonideal voltage source to a nonideal current source and solve the KCL problem (see Fig. 11.3b):

$$\frac{V_o}{R} = \left(\frac{1}{R} + sC\right)\hat{V}_C.$$

So the particular solution is

$$v_p(t) = 5 \text{ V}$$

and the homogeneous solution is

$$v_h(t) = V_0 e^{-t/RC}.$$

Using the principle of continuity of voltage across capacitors, and the "fact" that the capacitor voltage is zero when the switch is open, we arrive at

$$v_c(t) = 5(1 - e^{-t/RC}) \quad \text{(Volts)}.$$

This solution is not plotted because it is identical in form to the resistor voltage plotted in Fig. 11.2. The only difference is in the time constant, which is $\tau = RC$ for this circuit and $\tau = L/R$ for the previous case. How could we determine C from the oscilloscope data? This procedure is analogous to the previous one for the RL circuit…

Let's be a little careful about this "fact" that the initial voltage is zero. When the switch is open, there is no discharge path for the capacitor, so it remains charged. Thus, the first time that you try the circuit, it will probably work. However, if you open and close the switch a second time, most likely you won't trigger the oscilloscope, because the initial capacitor voltage will be too high. To avoid this problem, it is necessary to have a second switch that places a discharge resistor in parallel with the capacitor, as shown in Fig. 11.3c. It is necessary to close and open this switch before trying to repeat your measurement.

Now consider the RLC series circuit shown in Fig. 11.4. The current through the passive components (via KVL) is:

$$\hat{I} = V_o/(R + sL + \frac{1}{sC}).$$

It's more convenient to write this in terms of the capacitor voltage:

$$i = C\frac{dv_c}{dt} \Rightarrow V_c = \frac{I}{sC}.$$

Combining this last expression with the KVL equation yields:

$$\hat{V}_c = V_o/(s^2LC + sRC + 1).$$

Thus, the particular solution with the DC source is

$$V_p = 5\text{V}.$$

There are three possible homogeneous solutions depending on the sign of

$$D = \left(\frac{R}{2L}\right)^2 - \frac{1}{LC}.$$

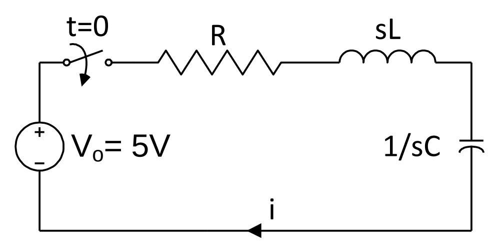

Figure 11.4 A simple RLC circuit.

If $D = 0$, $R = 2\sqrt{\frac{L}{C}}$ and the circuit is critically damped, so the homogeneous solution is:

$$v_h = Ae^{-\sigma t} + Bte^{-\sigma t} \text{ for } \sigma = \frac{R}{2L}.$$

The initial condition for the inductor is $i(0_+) = 0$, and this implies that $dV_c / dt(0_+) = 0$, but the capacitor voltage may not be discharged unless a discharge resistor is used as in Fig. 11.3c. If V_c $(0^+) = 0$, then

$$v_c(t) = 5\left[1 - e^{-\sigma t}(1+\sigma t)\right] \quad \text{Volts}$$

However, if $R > 2\sqrt{L/C}$, then $D > 0$ and the system is overdamped, with $v_h = Ae^{s_1 t} + Be^{s_2 t}$ for

$$s_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}.$$

Finally, if $R < 2\sqrt{\frac{L}{C}}$, $D < 0$, the system is underdamped and

$$v_h(t) = e^{-\sigma t}[A\cos(\omega t) + B\sin(\omega t)]$$

for σ as above and

$$\omega = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}.$$

In the lab you are supposed to design one circuit of each type. It's probably easiest to pick one inductor and one capacitor that are convenient, then choose three resistors: one that results in critical damping, one smaller, and one larger.

Now consider the parallel RLC circuit shown in Fig. 11.5a. After transforming the battery to a nonideal current source as in Fig. 11.5b we can use KCL to get

$$\frac{V_o}{R} = \left(\frac{1}{R} + \frac{1}{sL} + sC\right)\hat{V}_c = \left(\frac{1}{R} + \frac{1}{sL} + sC\right)\hat{V}_L.$$

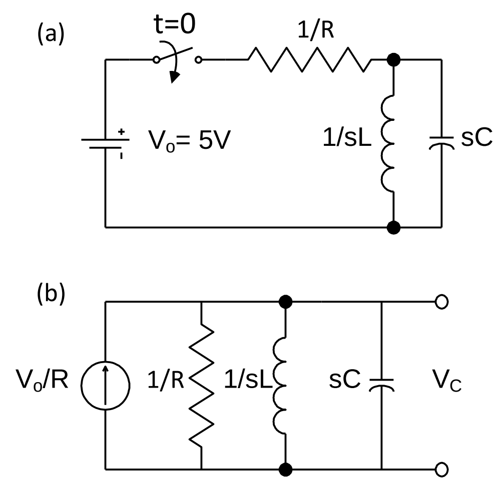

Figure 11.5 A parallel RLC circuit.

It's actually easier to solve for $i_L(t)$:

$$\hat{I}_L = V_o/(s^2LCR + sL + R).$$

So the particular solution is

$$i_p(t) = \frac{V_o}{R} = \frac{5}{R} \quad \text{(Amps)},$$

and the homogeneous roots are

$$s_{1,2} = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}\ .$$

There are three different types of solutions corresponding to critically damped, underdamped, and overdamped cases. The forms are the same as the forms shown for the RLC series case and are not repeated here. The dividing line between the three cases is different, however. For the RLC parallel case, the circuit is critically damped when $R = 0.5\sqrt{L/C}$. Furthermore, the circuit is underdamped whenever $R > 0.5\sqrt{L/C}$ and overdamped whenever $R < 0.5\sqrt{L/C}$. For the RLC series circuit, the solution was underdamped if the resistance was too low. In contrast, the RLC parallel circuit is underdamped if the resistance is too high. Can you explain the difference between the two cases in both mathematical and physical terms?

With the oscilloscope, we will measure the voltage across the LC circuit and not the current, so we must use the terminal relationship between the inductor voltage and current. Let's look at the critically damped case as a specific example. Now, the initial conditions at t=0 are that the current through the inductor is zero and the voltage across the capacitor is zero (no damping resistor is required – do you know why?) Thus, the total current through the inductor is:

$$i_L(t) = \frac{5}{R}\left[1-(1+\sigma t)e^{-\sigma t}\right] \quad \text{(A)} \qquad \text{for} \quad \sigma = \frac{1}{2RC}\ .$$

The inductor (and therefore capacitor) voltage is

$$v_L(t) = L\frac{di_L}{dt} = 5\frac{L}{R}\sigma^2 t e^{-\sigma t} = 10\sigma t e^{-\sigma t} \quad \text{Volts}\ .$$

Note that the voltage approaches zero as time goes to infinity. The maximum voltage can be found by setting the voltage derivative to zero:

$$V_{\max} = 10e^{-1}\,\text{V} \approx 3.68\,\text{V}$$

at the time

$$t_{\max} = 2RC.$$

Note that the maximum voltage is independent of R and L. Also note that if the oscilloscope trigger is set above 3.68V, the scope will never trigger when you pulse the circuit!

An example of an undamped LC series circuit is shown in Fig. 11.6. The reason for the diode will become evident later. Since the diode will be "on" while the capacitor is charging, we can just replace it with a short for now. We have already done the analysis, since it is the same as the RLC series circuit with $R = 0$. Taking into account zero initial capacitor voltage, the solution is:

$$v_c(t) = 5[1 - \cos(\omega t)] \quad \text{(V)}$$

for $\omega = \frac{1}{\sqrt{LC}}$.

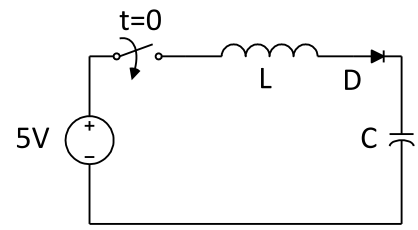

Figure 11.6 An undamped LC circuit.

The current through the capacitor is:

$$i_c(t) = C\frac{dv_c}{d\tau} = 5\sqrt{\frac{C}{L}}\sin(\omega t) \quad \text{(A).}$$

The current becomes zero when $\omega\tau = \frac{\tau}{\sqrt{LC}} = \pi$ or $\tau = \pi\sqrt{LC}$. At that point the diode turns off and the voltage on the capacitor stays constant. The maximum capacitor voltage is $v_c(\pi\sqrt{LC}) = 5(1 - \cos\pi) = 10\,\text{V}$. Thus, this circuit, known as a resonant charging circuit, charges the capacitor up to twice the battery voltage! (That's right, a damping resistor is necessary again for repeated measurements.)

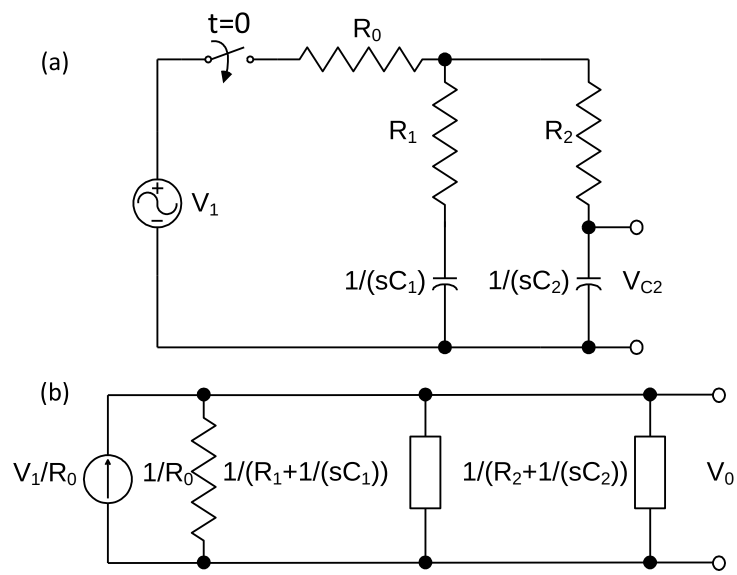

Figure 11.7 Another second order circuit.

As a final example, consider the two capacitor circuit shown in Fig. 11.7a. The transformation shown in Fig. 11.7b aids in the analysis of the circuit. The transfer function of the output voltage V_{C2} to the input voltage V_1 can be shown to be:

$$\frac{V_{C2}}{V_1} = \frac{1 + sR_1C_1}{s^2C_1C_2(R_0R_1 + R_0R_2 + R_1R_2) + s(C_2R_0 + C_1R_0 + C_1R_1 + C_2R_2) + 1}$$

The initial conditions, homogeneous solution, etc., can all be found by applying the principles you learned in ENEE 205.

A. Helpful Hints

1. Remember to discharge capacitors between "shots" in any circuit that doesn't naturally have a discharge path for the capacitor (e.g. series circuits).
2. Switch bouncing, which you learned about during some of the digital labs, can also cause a problem with analog transient circuits. The frequency of the switch bounce is critical. If the bounce period is much larger than the time constant of the circuit under test, you shouldn't have problems. If the period is much shorter, you may have problems taking measurements, but you should be able to view the qualitative transient characteristics. Adjusting the trigger hold-off may help, although it is likely that you will have to try each circuit many times to get a good shot, so always remember to discharge your capacitors.
3. The PSpice name for a switch that closes is sw_tClose; for a switch that opens: sw_tOpen.

Laboratory 10 Description - Transient Response in 1st and 2nd-Order Circuits

Objective:

To characterize transient signals in circuits with one or two energy-storing elements.

Pre-lab preparation

Part I – First-order circuits

1. Draw the wiring diagram for a switched RL circuit powered by a 5V battery.
2. Draw the wiring diagram for a switched RC circuit powered by a 5V battery.
3. Simulate the circuit in the previous step for 51 Ω and 0.47 μF. Plot the voltage across the capacitor as a function of time.

Part II – Second-order RLC circuits

4. Draw the wiring diagram for a switched RLC series circuit powered by a 5V battery. Given the available components, find RLC combinations that are overdamped, underdamped, and critically-damped (1 each).
5. Simulate the circuit in the previous step for the underdamped case. Plot the voltage across the capacitor as a function of time.
6. Draw the wiring diagram for a switched LC parallel circuit (with a series resistance R) powered by a 5V battery. Given the available components, find an RLC combination that is nearly critically-damped.
7. Simulate the circuit in the previous step. Plot the voltage across the capacitor versus time.

Experimental procedure:

Part I – First-order circuits

1. Construct the switched RL circuit with the 51 Ω resistor and the 2.2 mH inductor.
2. Operate the diligent oscilloscope in triggered mode and plot the voltage across the resistor.
3. Measure the time constant for the circuit.
4. Construct the switched RC circuit with the 1 kΩ resistor and the 0.47 µF capacitor.
5. Plot the voltage across the capacitor as a function of time.
6. Measure the time constant for the circuit.
7. Repeat the previous measurement with a 47 µF capacitor (but don't make more plots).

Part II – Second-order RLC circuits

8. Construct the switched RLC series circuit with the values you selected for the underdamped circuit.
9. Plot the voltage across the capacitor with time. (Make sure that you choose a time interval greater than the period of the oscillation so that you can attempt to measure the oscillation frequency.)
10. Repeat the above plot for the overdamped and critically-damped circuits.
11. Construct the switched RLC parallel circuit.
12. Plot the voltage across the capacitor with time.

Post-lab analysis:

Generate a lab report "following" the sample report available in Appendix A. Mention any difficulties encountered during the lab. Describe any results that were unexpected and try to account for the origin of these results (i.e. explain what happened). In ADDITION, answer the following questions/instructions:

Part I – First-order circuits

1. What was the inductance of the inductor according to the RL circuit measurements? Compare with the results from the ideal value and comment.
2. What was the capacitance of each capacitor according to the RC circuit measurements? Compare with the results from the ideal value and comment.

Part II – Second-order RLC circuits

3. Did all the RLC circuits behave as expected? If not, why not?

4. What was the oscillation frequency of the underdamped series RLC circuit?
5. What was the capacitor voltage overshoot of the underdamped series RLC circuit?
6. Which series RLC circuit approached steady-state most rapidly? Why?
7. What was the maximum voltage achieved on the capacitor in the parallel RLC circuit and how long after the switch closed did the maximum occur?
8. Compare the experimental and simulated results for both RLC circuits. Explain any differences that occurred.